## Exercise 136

For the following exercises, P is a point on the unit circle. a. Find the (exact) missing coordinate value of each point and b. find the values of the six trigonometric functions for the angle θ with a terminal side that passes through point P. Rationalize denominators.
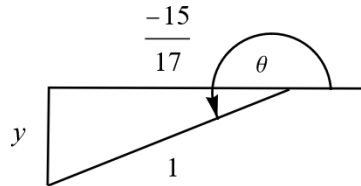
$$P\left(\frac{-15}{17}, y\right), \; y < 0$$

Solution

The given point P on the unit circle is shown below. $y < 0$ means that it's in the bottom half.
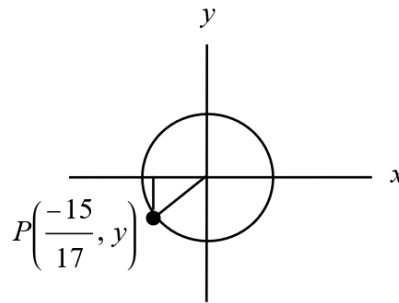
Zoom in on the right triangle formed by P. θ is the counterclockwise angle from the positive x-axis.

The hypotenuse has a length of 1 because P is on the unit circle. The sides of a right triangle are related by the Pythagorean theorem, and this allows us to determine y.

$$\left(\frac{-15}{17}\right)^2 + y^2 = 1^2$$

$$y^2 = 1^2 - \left(\frac{15}{17}\right)^2$$

$$y^2 = \frac{64}{289}$$

$$y = -\frac{8}{17}$$

Therefore, the six trigonometric functions are

$$\sin\theta = \frac{y}{1} = y = -\frac{8}{17}$$

$$\cos\theta = \frac{\frac{-15}{17}}{1} = -\frac{15}{17}$$

$$\tan\theta = \frac{y}{\frac{-15}{17}} = \frac{-\frac{8}{17}}{\frac{-15}{17}} = \frac{8}{15}$$

$$\csc\theta = \frac{1}{y} = -\frac{17}{8}$$

$$\sec\theta = \frac{1}{\frac{-15}{17}} = -\frac{17}{15}$$

$$\cot\theta = \frac{\frac{-15}{17}}{y} = \frac{\frac{-15}{17}}{-\frac{8}{17}} = \frac{15}{8}.$$